What happens when you **don't** have tools? The case of algebraic cryptanalysis

Léo Perrin[1]

[1]Inria, France

March 15, 2025

In this Presentation

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This comes not from my satisfaction with some tools...

... but from their absence*!

* at least at the time when it happened

Outline

Plan of this Section

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Securing Data

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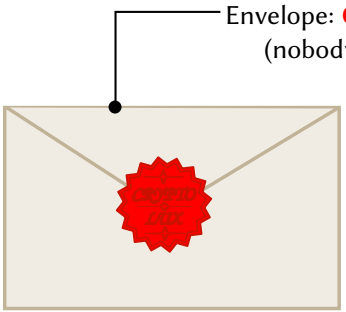

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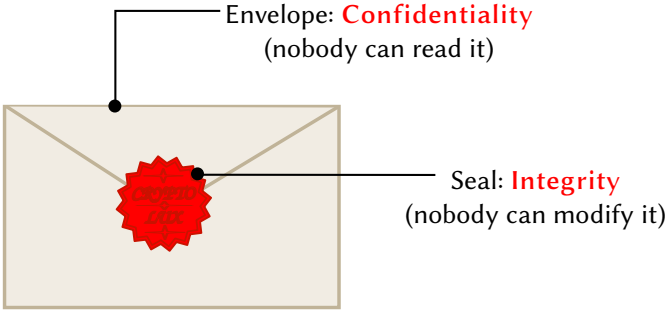

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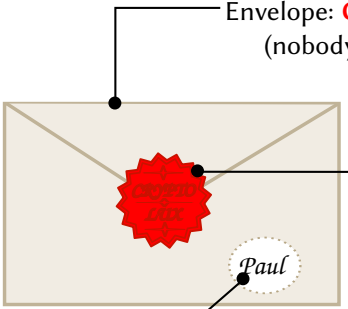

Securing Computation

More and more protocols intend to secure computations.

FHE Fully Homomorphic Encryption

MPC Multi Party Computations

ZK-* Zero Knowledge- [proof, argument...]

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These need (many) different types of symmetric primitives internally, and these are *often* "Arithmetization-Oriented".

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Example: Verifying if $y = c(ax + b)^{10} + x$ in R1CS

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2. $t_1 = t_0 + b$
3. $t_2 = t_1 \times t_1$
4. $t_3 = t_2 \times t_2$
5. $t_4 = t_3 \times t_3$
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- How to turn a computation into an arithmetic circuit depends on the operations allowed
- Its cost is also arithmetization-dependent—though low degree is usually welcome!

Symmetric Techniques for Advanced Protocols

MPC

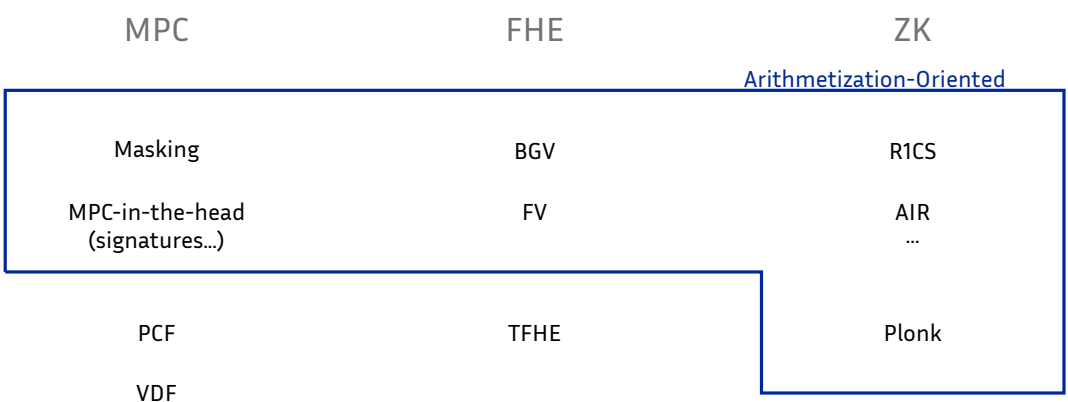
FHE

ZK

Symmetric Techniques for Advanced Protocols

MPC	FHE	ZK
Masking	BGV	R1CS
MPC-in-the-head (signatures...)	FV	AIR ...
PCF	TFHE	Plonk
VDF		

Symmetric Techniques for Advanced Protocols



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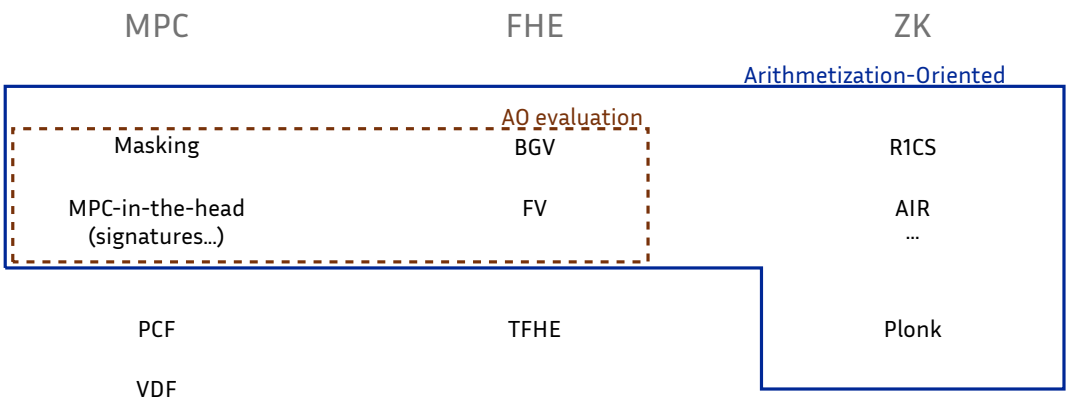

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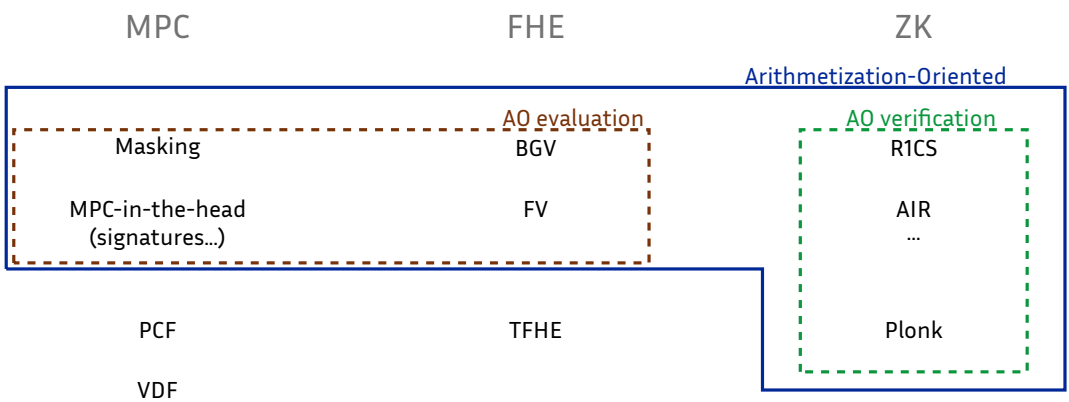

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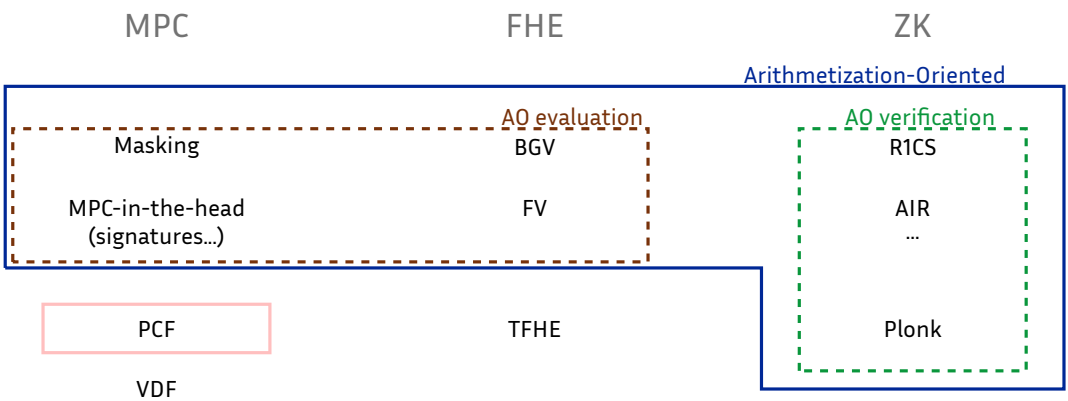

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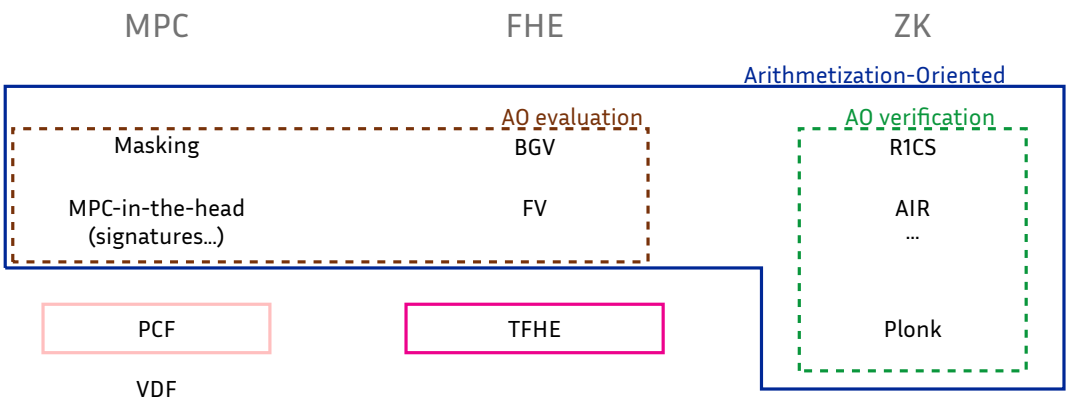

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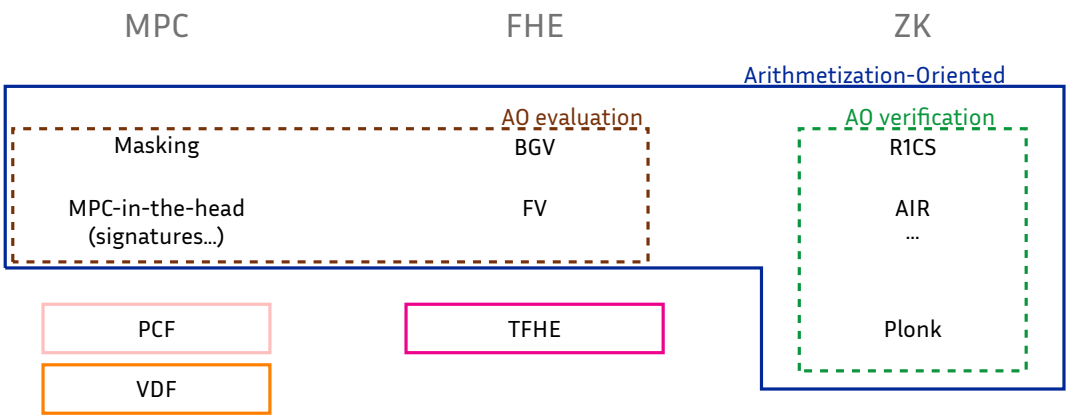

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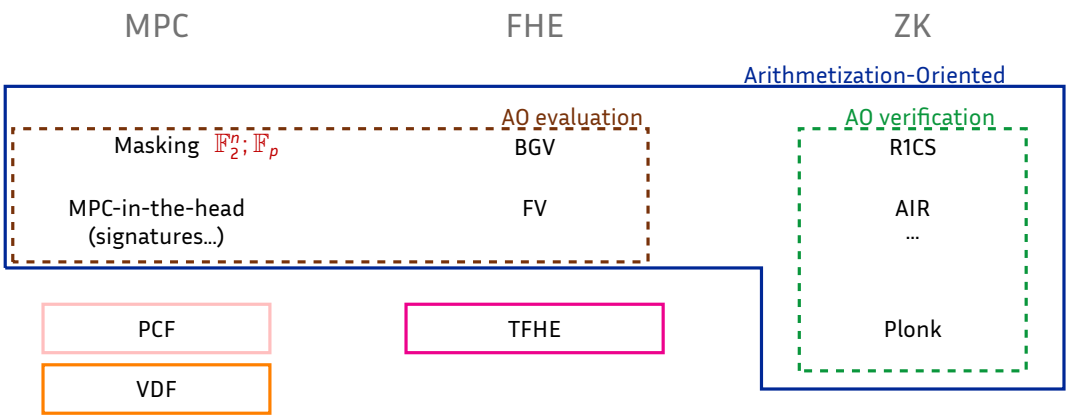

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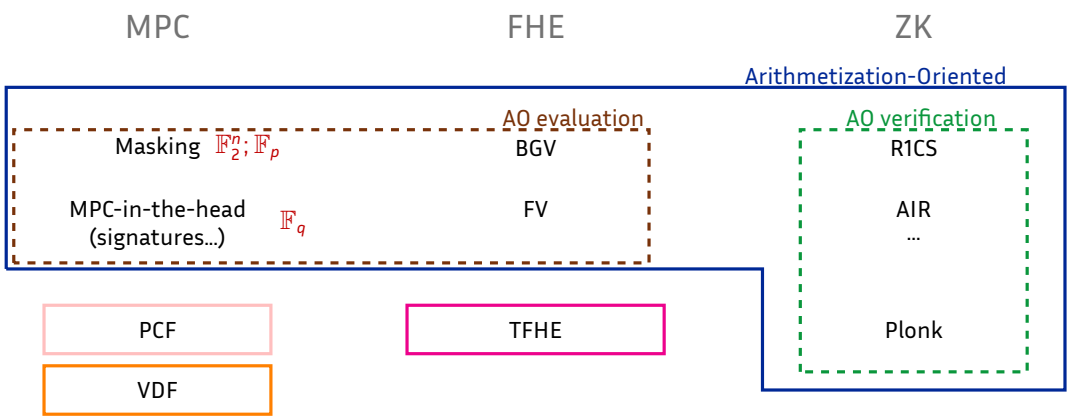

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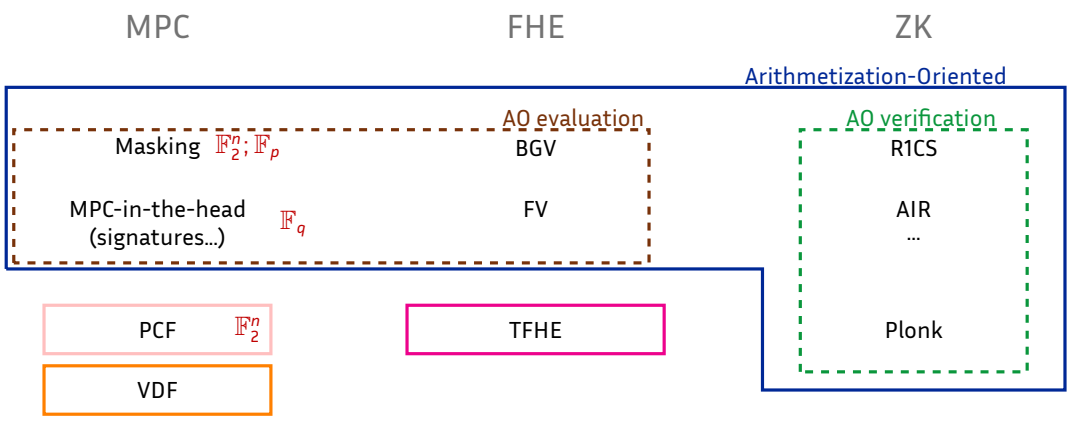

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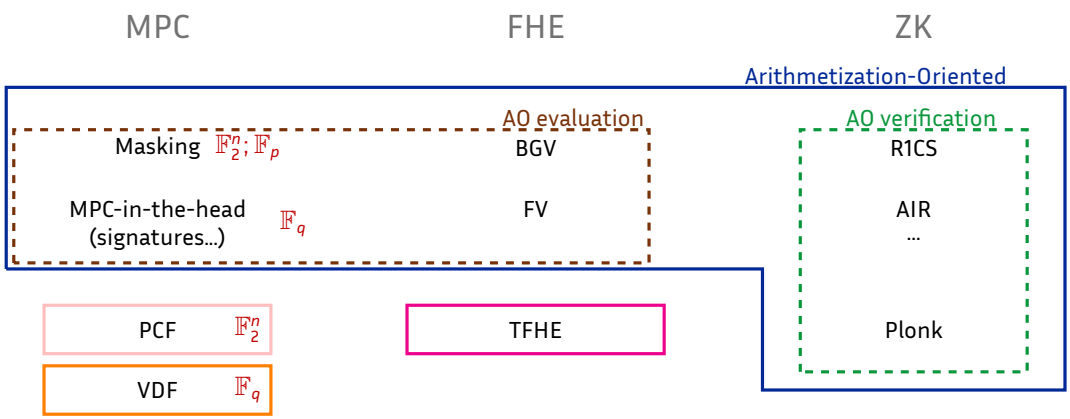

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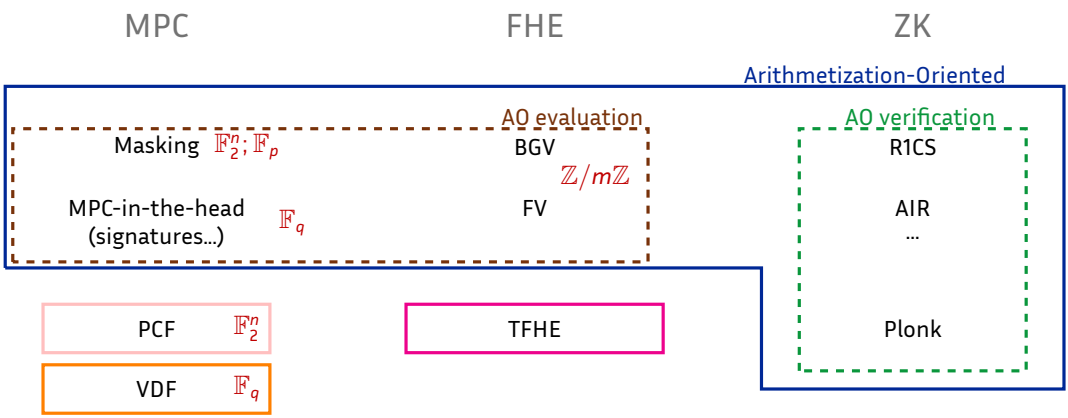

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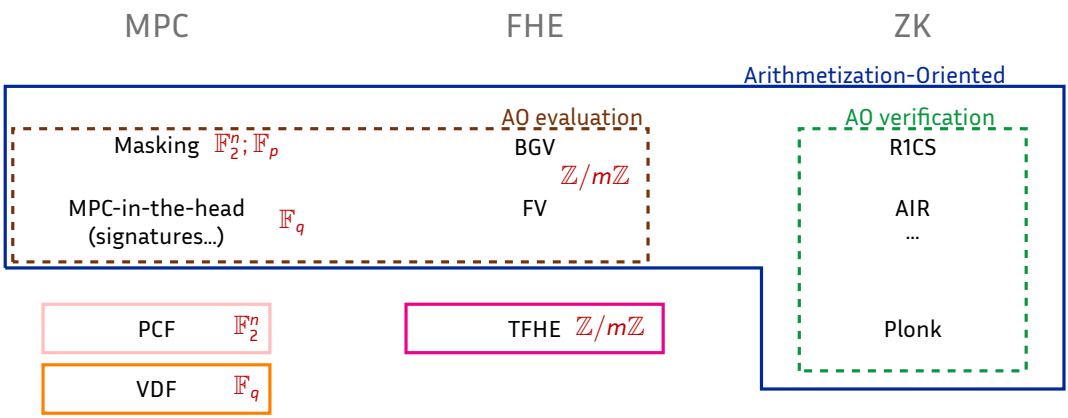

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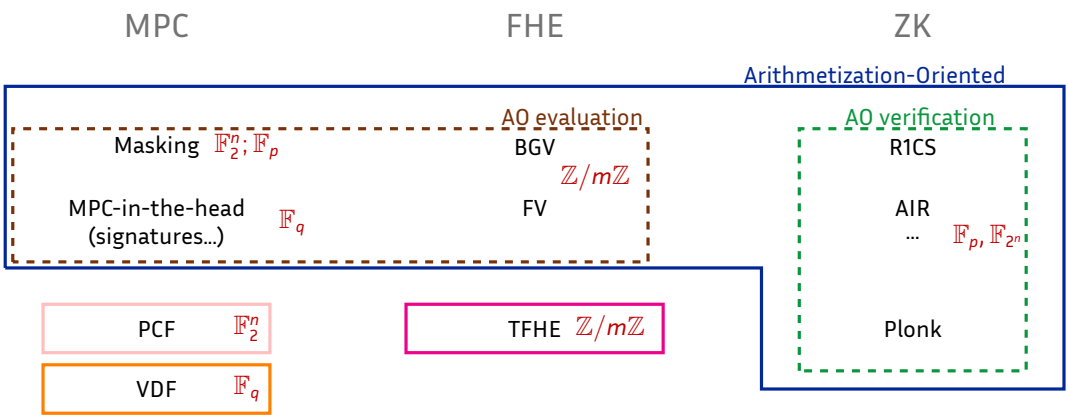

Arithmetization-Oriented Designs

What AO Often Means

The evaluation or the verification of the subfunctions involved must correspond to an arithmetic circuit with a low number of multiplications.

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- Big diffusion matrices
- For verif., $x \mapsto x^{1/d}$, d "small"

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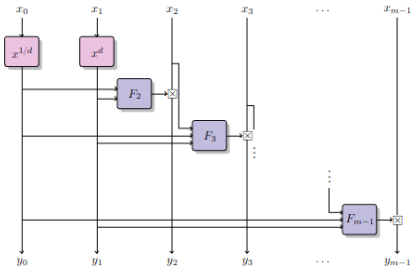
Example: Griffin



Source: PhD thesis of C. Bouvier, *Cryptanalysis and design of symmetric primitives defined over large finite fields*

Plan of this Section

Vulnerability to Algebraic Attacks

Arithmetization-Oriented Verification

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Vulnerability to Algebraic Attacks

There must exist a system of low degree equations that models an evaluation of the primitive.

Enabling a low degree arithmetization of verification implies a potential vulnerability to algebraic attacks!!

Generic System Solving: Steps and Tools

$$\begin{cases} p_1(x_1, \ldots, x_N) = 0 \\ \vdots \\ p_{k-1}(x_1, \ldots, x_N) = 0 \\ p_k(x_1, \ldots, x_N) = 0 \end{cases}$$

1. Define system

The Tools Used

SAGE Open source, mostly reliable...

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F4/F5 What makes it possible...

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Berlekamp-Rabin Easy to re-implement

The Freelunch Approach

Steps 2 and 3 are both computationally costly, but not for the same reasons. For most AOPs, step 2 dominates in practice...

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We can skip it!

Freelunch Crytpanalysis: Results

https://eprint.iacr.org/2024/347

Bariant, Augustin, et al. *The algebraic FreeLunch: Efficient Gröbner basis attacks against arithmetization-oriented primitives.* Annual International Cryptology Conference. Cham: Springer Nature Switzerland, 2024.

Name	α/e	Number of branches						
		2	3	4	5	6	8	≥ 12
Griffin	3	$\emptyset$	120 (16)	112 (15)	$\emptyset$	$\emptyset$	76 (11)	64 (10)
	5	$\emptyset$	141 (14)	110 (11)	$\emptyset$	$\emptyset$	81 (9)	74 (9)
Arion	3	$\emptyset$	128 (6)	134 (6)	114 (5)	119 (5)	98 (4)	$\emptyset$
	5	$\emptyset$	132 (6)	113 (5)	118 (5)	122 (5)	101 (4)	$\emptyset$
α-Arion	3	$\emptyset$	104 (5)	84 (4)	88 (4)	92 (4)	98 (4)	$\emptyset$
	5	$\emptyset$	83 (4)	87 (4)	91 (4)	94 (4)	101 (4)	$\emptyset$
Anemoi	3	118 (21)	$\emptyset$	-	$\emptyset$	-	-	-
	5	156 (21)	$\emptyset$	-	$\emptyset$	-	-	-
	7	174 (20)	$\emptyset$	-	$\emptyset$	-	-	-
	11	198 (19)	$\emptyset$	-	$\emptyset$	-	-	-

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Post-Mortem

From the designers' perspective

SAGE Did what it was supposed to do

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Berlekamp-Rabin	Did what it was supposed to do

Post-Mortem

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- Designers' relied on the assumption that the F4/F5 $\longrightarrow$ FGLM $\longrightarrow$ Univariate root finding chain was the **only** multivariate root-finding tool.
- Inefficient/proprietary software limited the designers' ability to prototype attacks.

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A better interface with another field

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- Underlying assumptions when using the F4/F5 $\longrightarrow$ FGLM $\longrightarrow$ Univariate root finding
- Deeper understanding of each step

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- Test in practical time the efficiency of different algorithms

A better interface with another field

- Underlying assumptions when using the F4/F5 $\longrightarrow$ FGLM $\longrightarrow$ Univariate root finding
- Deeper understanding of each step

We needed better tools!

Plan of this Section

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Lessons from the MILP and Root Finding Case

Root Finding

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Do we always use the "standard" solving method? Have we looked into a deeper integration?

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There is a "correct" level of abstraction that we must find

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Definition ("Tool")

A *tool* is a convenient abstraction layer above a specific know-how.

Plan of this Section

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- Suggest the best known algorithm for some problems

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Monotonous Ideal Degree Conjecture

A method to find a reasonable lower bound for the ideal degree.

`https://eprint.iacr.org/2024/605`

$$\left\{\begin{array}{l}\left.\begin{array}{l} x_0^{d_0} - P'_0(x_0, \ldots, x_{n-1}) \\ x_1^{d_1} - P'_1(x_0, \ldots, x_{n-1}) \\ \quad\ldots \\ x_{b-1}^{d_{b-1}} - P'_{b-1}(x_0, \ldots, x_{n-1}) \end{array}\right\} \text{basal part} \\ \\ P_b(x_0, \ldots, x_{n-1}) \\ P_{b+1}(x_0, \ldots, x_{n-1}) \\ \quad\ldots \\ P_{n-1}(x_0, \ldots, x_{n-1}) \end{array}\right.$$

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Conclusion

What is a Tool?

IMHO: *a convenient layer of abstraction above a specific know-how.*

Past Issues with Algebraic Attacks

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Thank you!